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Spacetime noncommutative effect on black hole as particle accelerators

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We study the spacetime noncommutative effect on black hole as particle accelerators and, find that particle falling from infinity with zero velocity cannot collide with unbound energy when the noncommutative Kerr black hole is exactly extremal. Our results also show that the bigger of the spinning black hole's mass is, the higher of center of mass energy that the particles obtain. For small and medium noncommutative Schwarzschild black hole, the collision energy depends on the black holes' mass.

Keywords: Noncommutative geometry; Black hole; Particle accelerators; Collision energy.

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1. Introduction

Bañados *et al*¹ and some other authors^{2-5,7} recently showed that particles falling freely from rest outside a Kerr black hole can collide with arbitrarily high center of mass energy in the limiting case of maximal black hole spin. They proposed that this might lead to signals from ultra high energy collisions, for example of dark matter particles. It seems that this arbitrarily high center of mass energy was generally accepted, however, it was criticized recently.⁶⁻⁹ The criticism related to very much issues, such as (1) an infinite time being taken to access the infinite collision energy in the extremal black hole case; (2) an infinite narrow strip between a horizon and a potential barrier where a particle can acquire the critical angular momentum due to multiple scattering in the nonextremal black hole case; (3) some astrophysical limitations such as gravitational radiation, backreaction, etc.. Jacobson *et al*⁷ pointed out that ultra-energetic collisions cannot occur in nature due to some practical limitations. Bejger *et al* and Harada *et al*^{8,9} pointed out that while the particle energy diverges, the position of the collision makes it impossible to escape to infinity and,

one shouldn't expect collisions around a black hole to act as spectacular cosmic accelerators.

In this paper we aim to show that when the quantum effect of gravity is considered, the presence of infinite collision energy cannot occur. The Planck energy scale is the realm where the general relativity will encounter with the quantum mechanics.^{10–12} So to the energetic collision particles, one should consider their quantum effect.

In the absence of a full quantum gravity theory, one usually uses effective theories to describe the quantum gravitational behavior such as Quantum Field Theory in Curved Spacetime. Noncommutative geometry is recently used as an effective tool for modelling the extreme energy quantum gravitational effects of the final phase of black hole evaporation, which are plagued by singularities at a semiclassical level. Quantum mechanics teach us that the emergence of a minimal length is a natural requirement when quantum features of phase space are considered. It also holds true to spacetime.^{11,13} The singularities in general relativity and ultraviolet divergences in quantum field theory can be avoided by the presence of spacetime minimal length. These singularities and divergences are nothing but pseudo effects due to the inadequacy of the formalism at small scales/extreme energies, rather than actual physical phenomena.

At very short distances the classical concept of spacetime should give way to a somewhat fuzzy picture.¹¹ The fundamental notion of the noncommutative geometry is that the picture of spacetime as a manifold of points breaks down at distance scales of the order of the Planck length: Spacetime events cannot be localized with an accuracy given by Planck length¹⁴ as well as particles do in the quantum phase space. So that the points on the classical commutative manifold should then be replaced by states on a noncommutative algebra and the point-like object is replaced by a smeared object¹⁵ to cure the singularity problems at the terminal stage of black hole evaporation.¹⁶

The approach to noncommutative quantum field theory follows two paths: one is based on the Weyl-Wigner- Moyal *-product and the other on coordinate coherent state formalism.¹⁵ In a recent paper,¹⁷ following the coherent state approach, it has been shown that Lorentz invariance and unitary, which are controversial questions raised in the *-product approach, can be achieved by assuming $\vartheta^{\mu\nu} = \vartheta \text{diag}(\epsilon_1, \dots, \epsilon_{D/2})$, where ϑ is a constant which has the dimension of length^2 , D is the dimension of spacetime¹⁸ and, there isn't any UV/IR mixing. Inspire by these results, various black hole solutions of noncommutative spacetime have been found.^{19,20} In this letter we use the noncommutative Kerr solution to study the problem of the particles' center of mass energy when they collide at the horizon.

2. The noncommutative Kerr black hole

In history, the study on rotating black hole solution had been met with technical difficulties in solving Einstein equations, and completely ignored the appropriate matter source. The obtainment of Kerr solution is based on the so-called “vacuum solution” method consisting in assuming an additional symmetry for the metric and solving field equations with no matter source. Integration constants are then determined comparing the weak field limit of the solution with known Newtonian-like forms. This approach is physically unsatisfactory especially in General Relativity, where basic postulate is that geometry is determined by the mass-energy distribution. Using the noncommutative geometry method and following the basic Einstein’s idea that spacetime is curved due to the presence of matter, Smalagic *et al* derives the line element of the noncommutative Kerr black hole²⁰

$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right) dt^2 - \frac{4Mra \sin^2 \theta}{\rho^2} dt d\phi + \frac{\rho^2 dr^2}{\Delta} + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2Mra^2 \sin^2 \theta}{\rho^2}\right) \sin^2 \theta d\phi^2, \quad (1)$$

and

$$M = \frac{2M_0}{\sqrt{\pi}} \gamma\left(\frac{3}{2}, \frac{r^2}{4\vartheta}\right), \quad \gamma\left(\frac{3}{2}, x\right) \equiv \int_0^x t^{1/2} e^{-t} dt, \\ \rho^2 = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 - 2Mr + a^2, \quad (2)$$

where ϑ is a spacetime noncommutative parameter^a, a is the spinning black hole’s angular momentum. The commutative Kerr metric is obtained from (1) in the limit $r/\sqrt{\vartheta} \rightarrow \infty$. Equation (1) leads to the mass distribution $M(r) = 2M_0 \gamma(3/2, r^2/4\vartheta)/\sqrt{\pi}$, where M_0 is the total mass of the source. In the classical General Relativity, black hole’s mass is dealt by point-like mass, and then it can be a constant. But in the noncommutative gravity, the mass cannot be treated as a constant, it is the mass distribution $M(r)$.

Depending on the values of a , $\sqrt{\vartheta}$ and M_0 , the metric displays different causal structure: existence of two horizons (non-extremal black hole), one horizon (extremal black hole) or no horizons (massive spinning droplet). Due to $\Delta(r_+) = 0$ cannot be solved analytically, we list some values of the maximum angular momentum a_{\max} , the single horizon r_+ and mass distribution M_+ on the horizon in Table 1 by letting $M_0 = 1$ ^b.

Table 1 shows that the maximum angular momentum a_{\max} decreases with the increase of the spacetime noncommutative parameter $\sqrt{\vartheta}$. It indicates the restriction of the spacetime non-commutativity on the angular momentum of black hole

^aThe notation ϑ used here is a constant as well as Plank constant \hbar , but we still call it a spacetime noncommutative parameter since it up to now is undetermined.

^bThe units we used here and hereafter is the total mass of the black hole M_0 , i.e., $\frac{r}{M_0} \rightarrow r$, $\frac{a}{M_0} \rightarrow a$, $\frac{\sqrt{\vartheta}}{M_0} \rightarrow \sqrt{\vartheta}$.

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Table 1. Numerical values for the radius of the single event horizon and the mass distribution on the horizon in the extremal spinning noncommutative black hole spacetime with different $\sqrt{\vartheta}$ and a_{\max} ($M_0 = 1$).

$\sqrt{\vartheta}$	0.525177	0.52517	0.525	0.52	0.48	0.44	0.40	0.36
a_{\max}	0	0.00589	0.029159	0.15742	0.45983	0.62170	0.73892	0.82841
r_+	1.58826	1.58749	1.58727	1.58460	1.54842	1.49613	1.43401	1.36328
M_+	0.79413	0.79376	0.79390	0.80012	0.84287	0.87724	0.90738	0.93336
$\sqrt{\vartheta}$	0.32	0.28	0.24	0.20	0.16	0.12	0.08	0.04
a_{\max}	0.89656	0.94621	0.97876	0.99539	0.99979	0.999998	$1 - 10^{-14}$	1.00000
r_+	1.28295	1.20207	1.11883	1.04683	1.00638	1.00035	1.000000141	1.00000
M_+	0.95475	0.97344	0.987528	0.99665	0.99981	$1 - 10^{-7}$	1.00000	1.00000

which implies that i) if $\sqrt{\vartheta}$ is strong, its single horizon is close to that of the non-commutative Schwarzschild black hole; ii) if $\sqrt{\vartheta}$ is weak, its single horizon is close to that of the commutative Kerr hole. In other words, the point-like structure of spacetime lets $a \leq M_0$, while the minimal length of spacetime leads to $a < M_0$.

When $M_0 > 1.90412\sqrt{\vartheta}$ and $0 \leq a < a_{\max}$, the two horizons (non-extremal black hole) are given by

$$r_{\pm}^2 = \frac{4r_{\pm}}{\sqrt{\pi}} \gamma(3/2, r_{\pm}^2/4\vartheta) - a^2. \quad (3)$$

which is different from the commutative Kerr black hole. The line element (1) describes the geometry of a noncommutative black hole and should give us useful insights about possible spacetime noncommutative effects on particle accelerators.

3. Near horizon collision in extremal noncommutative Kerr black hole spacetime

The solution to the geodesic equation of the noncommutative Kerr black hole is given by

$$\begin{aligned} \frac{dt}{d\tau} &= -\frac{2Mr a L + a^2 E \Delta - E(r^2 + a^2)^2}{r^2 \Delta}, \\ \frac{dr}{d\tau} &= \pm \frac{\sqrt{2Mr(L - aE)^2 + 2Mr^3 E^2 - r^2 L^2 + \Delta r^2 (E^2 - m^2)}}{r^2}, \\ \frac{d\phi}{d\tau} &= -\frac{(a^2 - \Delta)L - 2Mr a E}{r^2 \Delta}, \end{aligned} \quad (4)$$

where E , L , m are the particle's energy, angular momentum and rest mass. We assume throughout the paper that the motion of particles occur in the equatorial plane.

Firstly, we should find the range of angular momentum of particles which can reach to the horizon under the condition $a = a_{\max}$. The maximum/minimum angular momentum of particles can be found using the effective potential for the radial motion in the equatorial plane. The proper time derivative of the (Boyer-Lindquist)

radial coordinate of orbital motion satisfies $\dot{r}^2/2 + V_{\text{eff}}(r, L, \sqrt{\vartheta}) = 0$, where the effective potential is given in terms of the angular momentum L by

$$V_{\text{eff}} = -\frac{Mm^2}{r} + \frac{L^2 - a^2(E^2 - m^2)}{2r^2} - \frac{M(L - aE)^2}{r^3} - \frac{E^2 - m^2}{2}. \quad (5)$$

The maximum/minimum angular momentum we are looking for is defined by $V_{\text{eff}} = dV_{\text{eff}}/dr = 0$. The numerical values of maximum/minimum angular momentum are listed in Table 2, and some effective potentials of particles with the critical, super-critical and maximum angular momentum are showed in Fig. 1.

Secondly, we should find the critical angular momentum of particles whose center of mass energy E_{cm} were assumed to be arbitrary high when $a = a_{\text{max}}$. On the background metric (1), the CM energy of two particles 1 and 2 is⁵

$$\frac{E_{\text{cm}}^2}{2} = m^2 + E_1 E_2 + \frac{F(r) - G(r)}{D(r)}, G(r) = 2\sqrt{-V_{\text{eff}1}}\sqrt{-V_{\text{eff}2}}, D(r) = \frac{a^2}{r^2} - \frac{2M}{r} + 1, \\ F(r) = 2\left[\frac{a^2}{r^2}\left(1 + \frac{M}{r}\right) + \left(1 - \frac{M}{r}\right)\right]E_1 E_2 - \frac{2Ma}{r^3}(E_1 L_2 + E_2 L_1) - \left(1 - \frac{2M}{r}\right)\frac{L_1 L_2}{r^2}, \quad (6)$$

where two particles' mass $m_1 = m_2 = m$. It is believed that if the collision occurs near the horizon, the CM energy can be unboundedly high. So the behavior of formula (6) near horizon should be considered. Here we would like to use Zaslavskii's formula³ for seeking E_{cm} in a model-independent form

$$\left(\frac{E_{\text{cm}}^2}{2m^2}\right)_H = 1 + \frac{b_{1H}(L_{2H} - L_2)}{2(L_{1H} - L_1)} + \frac{b_{2H}(L_{1H} - L_1)}{2(L_{2H} - L_2)} - \frac{L_1 L_2}{(g_{\phi\phi})_H}, \\ L_{iH} = \frac{E_i}{\omega_H}, \quad b_{iH} = 1 + \frac{L_{iH}^2}{(g_{\phi\phi})_H}, \quad (7)$$

where $\omega_H = (-g_{t\phi})_H/(g_{\phi\phi})_H$, $L = lmM_0$. After taking $E_1 = E_2 = E$ for simplicity, we obtain the center of mass energy for the collision:

$$\left(\frac{E_{\text{cm}}}{2m}\right)_H = \sqrt{1 + \frac{b_H(l_1 - l_2)^2}{4(l_H - l_1)(l_H - l_2)}}. \quad (8)$$

Then the critical angular momentum l_H of particles whose center of mass energy E_{cm} were assumed to be arbitrary high when $a = a_{\text{max}}$ can be found via

$$l_H = \frac{E}{\omega_H} = \frac{r_+(r_+^2 + a^2) + 2M_+ a}{2M_+ a}. \quad (9)$$

The numerical values of critical angular momentum are listed in Table 2.

From Table 2, one can see that all the critical angular momentum lies beyond the range $(l_{\text{min}}, l_{\text{max}})$, which shows that the unlimited center of mass energy cannot be approached. In addition, it is interesting that $l_{\text{min}} = -l_{\text{max}} = -4.0$, $l_H = \infty$ when $\sqrt{\vartheta}$ is maximum, which is the same as that of the commutative Schwarzschild black hole; $l_{\text{min}} = -4.82843$, $l_{\text{max}} = l_H = 2.0$ when $\sqrt{\vartheta} \rightarrow 0$, which is the same as that of the commutative Kerr black hole. Fig. 1 shows that the particle effective

Table 2. Numerical values for the maximum, minimum and critical angular momentum in the extremal spinning noncommutative black hole spacetime with different $\sqrt{\vartheta}$ and a_{\max} with $M_0 = 1, m = 1, E = 1$.

$\sqrt{\vartheta}$	0.525177	0.52517	0.525	0.52	0.48	0.44	0.40	0.36
l_{\max}	4.0	3.99409	3.97061	3.83578	3.46907	3.22759	3.01662	2.81895
l_{\min}	-4.0	-4.00587	-4.02894	-4.15167	-4.41647	-4.54692	-4.63736	-4.70437
l_H	∞	427.8693	86.4322	16.1080528	5.6739587	4.222171	3.52188779	3.07190361
$\sqrt{\vartheta}$	0.32	0.28	0.24	0.20	0.16	0.12	0.08	0.04
l_{\max}	2.62738	2.4382	2.24999	2.09103	2.011	2.0005	2.0000002	2.00000
l_{\min}	-4.75431	-4.79013	-4.81337	-4.82517	-4.82828	-4.82843	-4.82843	-4.82843
l_H	2.732436	2.47333205	2.2576964	2.09631272	2.0128107	2.0007053	2.00000028	2.00000

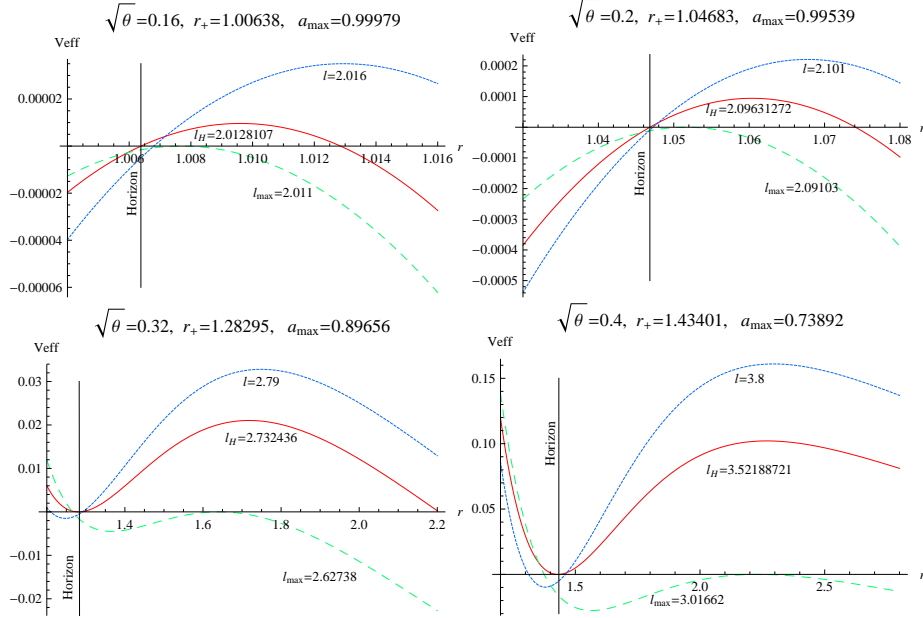


Fig. 1. The effective potentials of particles with critical, super-critical and maximum angular momentum in the extremal spinning noncommutative black hole spacetime with different $\sqrt{\vartheta}$ and a_{\max} with $M_0 = 1, m = 1, E = 1$.

potentials with critical angular momentum is positive near the horizon, so they cannot approach to the horizon.

With these data, we obtain E_{cm} for noncommutative Kerr black holes with $l_1 = l_{\min}$, $l_2 = l_{\max}$, $a = a_{\max}$ by using Eq. (8) with

$$b_H = 1 + \frac{l_H^2}{(g_{\phi\phi})_H} = 1 + \frac{r_+ [r_+ (r_+^2 + a^2) + 2M_+ a^2]}{(2M_+ a)^2}, \quad (10)$$

and list them in Table 3.

From Table 3, one can see that, for the noncommutative Kerr black hole case,

Table 3. Numerical values for the center of mass energy of particles colliding at the horizon in the extremal spinning noncommutative black hole spacetime with different $\sqrt{\vartheta}$ and a_{\max} with $M_0 = 1, E = m = 1$.

$\sqrt{\vartheta}$	0.525177	0.52517	0.525	0.52	0.48	0.44	0.40	0.36
E_{cm}	5.41948	5.42186	5.42481	5.50061	6.20380	7.14606	8.46939	10.5215
$\sqrt{\vartheta}$	0.32	0.28	0.24	0.20	0.16	0.12	0.08	0.04
E_{cm}	14.6674	23.2222	46.0119	52.5201	87.1905	258.789	12962.9	∞

the center of mass energy of particles is bounded by using a more generic formula in a model-independent form.

Here we can see the spacetime noncommutative effect on black hole as particle accelerators. The spacetime noncommutative effects avoid the presence of infinite collision energy via preventing the black hole's angular momentum to reach to the black hole's mass, $a = M_0$. In other words, the point-like structure of spacetime lets $E_{\text{cm}} \leq \infty$, while the presence of spacetime minimal length leads to $E_{\text{cm}} < \infty$.

From Table 3, one can also see that, the bounded E_{cm} increases with the spacetime noncommutative parameter decreasing and, if $\sqrt{\vartheta} \rightarrow 0$, it coincides with that of the commutative case.

If we choose the spacetime noncommutative constant $\sqrt{\vartheta} = 1$ units^c, then decrease of $\sqrt{\vartheta}$ in Table 3 is corresponding to increase of black hole mass M_0 . It can be easily seen from Table 4. $\sqrt{\vartheta} \rightarrow 0$ is corresponding to $M_0 \rightarrow \infty$. Therefore E_{cm} increases with the black hole's mass increasing, and it cannot be approached to arbitrary high unless the black hole mass is infinite. This can be easily seen from Table 4 which is related to Table 1 by take the mass parameter M_0 in the place of $\sqrt{\vartheta}$.

Table 4. Numerical values for the radius of the single event horizon in the extremal spinning noncommutative black hole spacetime with different M_0 and a_{\max} with $\sqrt{\vartheta} = 1$.

M_0	1.90412	2.3	2.7	3.1	3.5	3.9	4.3	4.7
a_{\max}	0	1.46963	2.18007	2.76750	3.29068	3.77295	4.22698	4.66100
r_+	3.02343	3.42096	3.7305	3.99299	4.24353	4.48729	4.73766	5.00025
M_0	5.1	5.5	5.9	6.3	6.7	7.1	7.5	7.9
a_{\max}	5.08100	5.49169	5.58677	6.29888	6.69965	7.09990	7.49997	7.89999
r_+	5.28989	6.60919	5.95209	6.32217	6.71019	7.10614	7.50523	7.90399

In Table 3, when $\sqrt{\vartheta} = 0.525177$, $a = 0$, $l_H = \infty$, then Eq. (8) is invalid any more. We use another formula to seek E_{cm} for noncommutative Schwarzschild black

^cThe units we used here is the spacetime noncommutative constant $\sqrt{\vartheta}$, i.e. $\frac{r}{\sqrt{\vartheta}} \rightarrow r$, $\frac{a}{\sqrt{\vartheta}} \rightarrow a$, $\frac{M_0}{\sqrt{\vartheta}} \rightarrow M_0$.

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holes with $l_1 = l_{\min}$, $l_2 = l_{\max}$, $a = 0$ ¹

$$\left(\frac{E_{\text{cm}}}{m}\right)_H^2 = 2\frac{H'}{(r\Delta)'}\Big|_{r \rightarrow r_+}, \quad H = (2M - r)l_1l_2 - 2Ma(l_1 + l_2) + 2r(r^2 + a^2) \\ + 2M(a^2 - r^2) - \sqrt{2M(l_1 - a)^2 + 2Mr^2 - rl_1^2}\sqrt{2M(l_2 - a)^2 + 2Mr^2 - rl_2^2}, \quad (11)$$

where the prime ' denotes d/dr .

Table 5. Numerical values for the center of mass energy of particles colliding at the horizon in the noncommutative Schwarzschild black hole spacetime with different $\sqrt{\vartheta}$ with $M_0 = 1, E = m = 1$.

$\sqrt{\vartheta}$	0.525177	0.52517	0.525	0.52	0.48	0.44	0.40	0.36
E_{cm}	5.41948	5.40702	5.34855	5.07212	4.65865	4.54187	4.49482	4.47757
$\sqrt{\vartheta}$	0.32	0.28	0.24	0.20	0.16	0.12	0.08	0.04
E_{cm}	4.47290	4.47218	4.47214	4.47214	4.47214	4.47214	4.47214	4.47214

From Table 5, one can see that, for the noncommutative Schwarzschild black hole case, the bounded E_{cm} increases with the spacetime noncommutative parameter if $0.52517 < \sqrt{\vartheta} \leq 0.24$ which is different from commutative case. If $0 < \sqrt{\vartheta} < 0.24$, it coincides with that of the commutative case, i.e. the collision energy does not depend on the mass of black hole.¹

4. ISCO particle collision in extremal spinning noncommutative black hole spacetime

Harada *et al*⁵ pointed out that either a particle plunging from the ISCO (innermost stable circular orbit) to the horizon or orbiting the ISCO collides with another particle can obtain an arbitrarily high CM energy without any artificial fine-tuning in an astrophysical context. In this section, we consider these collisions in the extremal spinning noncommutative black hole spacetime.

The ISCO in the Kerr spacetime is explicitly given by Bardeen, Press and Teukolsky.²¹ The circular orbit on the equatorial plane is given by $V_{\text{eff}}(r) = 0$, $V'_{\text{eff}}(r) = 0$, and the ISCO is determined by the condition $V''_{\text{eff}}(r) = 0$. Here we consider only the prograde ISCO, and the numerical values for the radius of the prograde ISCO r_p , the particle energy E_p , angular momentum l_p are listed in Table 6, some effective potentials are showed in Fig. 2.

From Table 6, we can see that when noncommutative parameter $\sqrt{\vartheta} \rightarrow 0$, the prograde ISCO radius $r_p \rightarrow M_0$, particle energy and angular momentum $E_p \rightarrow m/\sqrt{3}$, $L_p \rightarrow 2mM_0/\sqrt{3}$. When noncommutative parameter $\sqrt{\vartheta} \rightarrow 0.525177$, the prograde ISCO radius $r_p \rightarrow 6M_0$. These results coincides with those ones obtained in commutative case.

With these data, the CM energy of an ISCO particle collision can be obtained. As for the case that a particle plunging from the ISCO collides with another one,

Table 6. Numerical values for the radius of the prograde ISCO r_p , the particle energy E_p angular momentum l_p in the extremal spinning noncommutative black hole spacetime with different $\sqrt{\vartheta}$ and $a_{\max}(M_0 = 1, m = 1)$.

$\sqrt{\vartheta}$	0.525177	0.52517	0.525	0.52	0.48	0.44	0.40	0.36
r_p	6.0	5.98075	5.90443	5.47538	4.38851	3.7374	3.21036	2.75103
E_p	0.942809	0.94262	0.94185	0.93715	0.92092	0.90644	0.89017	0.87057
l_p	3.4641	3.45854	3.43639	3.30874	2.95718	2.72145	2.51181	2.31115
$\sqrt{\vartheta}$	0.32	0.28	0.24	0.20	0.16	0.12	0.08	0.04
r_p	2.33639	1.9544	1.59737	1.26164	1.03582	1.00628	1.0000002	1.00000
E_p	0.84581	0.81306	0.76739	0.69992	0.60947	0.58099	0.5773504	0.5773503
l_p	2.11120	1.90599	1.68933	1.45464	1.2237	1.16201	1.1547008	1.1547007

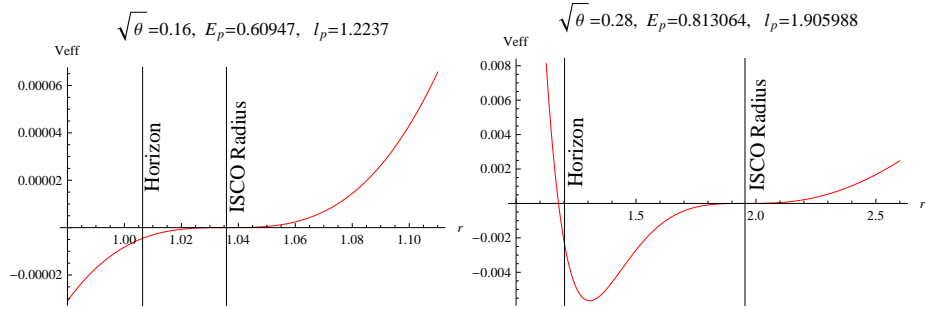


Fig. 2. The effective potentials of the prograde ISCO particles in the extremal spinning noncommutative black hole spacetime with different $\sqrt{\vartheta}$ ($M_0 = 1, m = 1$).

we assume that $E_1 = E_p, l_1 = l_p, E_2 = 1, l_2 = l_{\min}$ and use the formula (7) to obtain CM energy. The numerical values are listed in Table 7. It is easy to see that it cannot collide with arbitrary high CM energy.

Table 7. Numerical values for the center of mass energy of particle plunging from the prograde ISCO colliding with other particle near the horizon in the extremal spinning noncommutative Kerr black hole spacetime with different $\sqrt{\vartheta}$ ($l_1 = l_p, l_2 = l_{\min}, M_0 = 1, m = 1$).

$\sqrt{\vartheta}$	0.525177	0.52517	0.525	0.52	0.48	0.44	0.40	0.36
E_{cm}	4.14213	4.15491	4.43082	4.84061	5.56238	6.93147	7.84845	8.58937
$\sqrt{\vartheta}$	0.32	0.28	0.24	0.20	0.16	0.12	0.08	0.04
E_{cm}	9.99493	12.2423	17.1973	29.1802	55.6678	155.734	7478	35178.1

As for the case that a particle orbiting on the ISCO and collides with another one, we assume that $E_1 = E_p, l_1 = l_p, E_2 = 1, l_2 = l_{\min}$ and use the formula (6) with $r = r_p$ to obtain CM energy. The numerical values are listed in Table 8. It is easy to see that it cannot also collide with arbitrary high CM energy.

Table 8. Numerical values for the center of mass energy of particle on the prograde ISCO colliding with other particle near the horizon in the extremal spinning noncommutative Kerr black hole spacetime with different $\sqrt{\vartheta}$ ($l_1 = l_p, l_2 = l_{min}, M_0 = 1, m = 1$).

$\sqrt{\vartheta}$	0.525177	0.52517	0.525	0.52	0.48	0.44	0.40	0.36
E_{cm}	2.36601	2.36781	2.4048	2.42282	2.59655	2.76469	2.96751	3.2357
$\sqrt{\vartheta}$	0.32	0.28	0.24	0.20	0.16	0.12	0.08	0.04
E_{cm}	3.6945	4.33574	5.3927	8.1807	22.9037	48.9025	6606.5	30173.4

5. Near horizon collision in nonextremal noncommutative Kerr black hole spacetime

For the nonextremal horizon, A particle with $E = 1$ cannot penetrate from infinity to the horizon but, nonetheless, there is a narrow region between a horizon and a potential barrier where such motion can occur that can generate acceleration to arbitrary large energies^{2,3}

$$0 \leq r - r_+ \leq r_{max}, \quad (12)$$

where

$$r_{max} = \frac{\varepsilon^2}{b_H(N^2)'(r_+)}, \quad N^2 = \frac{g_{t\phi}^2}{g_{\phi\phi}} - g_{tt} = \frac{\Delta}{r^2 + a^2 + 2Ma^2/r}, \quad \varepsilon = 1 - \frac{l}{l_H}. \quad (13)$$

Some numerical values of (r_{max}, E_{cm}) for noncommutative Kerr black hole and (r'_{max}, E'_{cm}) for commutative Kerr black hole are listed in Table 9.

Table 9. Numerical values of (r_{max}, E_{cm}) for noncommutative Kerr black hole and (r'_{max}, E'_{cm}) for commutative Kerr black hole with $a = a_{max}(1 - 0.01)$, $l_1 = l_H(1 - 0.01)$, $l_2 = l_{min}$ and Eq. (8) with $M_0 = 1, m = 1$.

$\sqrt{\vartheta}$	0.36	0.32	0.28	0.24	0.20	0.16	0.12	0.08
$r_{max} \times 10^{-4}$	2.72803	3.19306	3.74738	4.47368	5.40021	6.03336	6.08874	6.08881
$r'_{max} \times 10^{-4}$	0.74769	1.17096	1.85710	3.04582	4.88076	6.01662	6.08874	6.08881
E_{cm}	31.2842	29.5185	28.2908	27.4611	26.8900	26.8264	26.8180	26.8180
E'_{cm}	32.1719	30.0384	28.5690	27.5252	27.0018	26.8268	26.8180	26.8180

From table 9, it is easy to see the region between a horizon and a potential barrier is infinite narrow where a particle can acquire the critical angular momentum due to multiple scattering. It is also shows that the spacetime noncommutative effect constrains this E_{cm} also in nonextremal rotating black hole spacetime.

6. Summary

We have examined the mechanism that using spinning and non-spinning black holes as particle accelerators in presence of quantum effect of gravity. Our results show that infinite center of mass energy for the colliding particles cannot be attained

unless the mass of the black hole is infinite. This is due to that the point-like structure of spacetime lets $a \leq M_0$, and $E_{\text{cm}} \leq \infty$; while the presence of spacetime minimal length leads to $a < M_0$, and $E_{\text{cm}} < \infty$.

The present mechanisms that prevent infinite energies are (1) an infinite time being taken to access the infinite collision energy in the extremal black hole case; (2) an infinite narrow strip between a horizon and a potential barrier where a particle can acquire the critical angular momentum due to multiple scattering in the nonextremal black hole case; (3) some astrophysical limitations such as gravitational radiation, backreaction, etc. So one can see that the quantum effect of gravity is an other preventing mechanism.

Additionally, for noncommutative rotating black hole, the collision energy increases with the increasing of black hole's mass as the black hole is exactly extremal. For noncommutative Schwarzschild black hole, the bound E_{cm} decreases with the black hole mass if $0.52517 < \sqrt{\theta} \leq 0.24$ which is different from commutative case and, if $0 < \sqrt{\theta} < 0.24$, the collision energy does not depend on the mass of black hole which coincides with that of the commutative case.

Gamma-ray bursts and ultra-high-energy cosmic rays provide an important testing ground for fundamental physics. Some cosmic rays have been observed with extremely high energies. These rays may be generated at the black hole horizon which acts as a particle accelerator. Our study shows that in order to acquire higher energy, the bigger of black hole mass is required. It can be used to explore new ideas in the structure of spacetime at short (Planckian) distance scale.

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